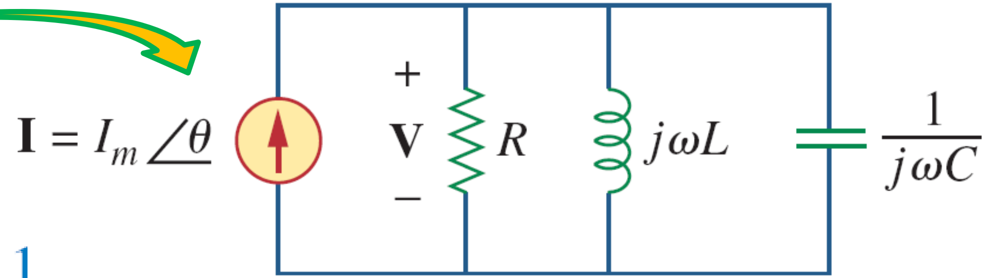
Lecture – 17 **Date: 09.10.2017**

- Parallel Resonance
- Active and Passive Filters

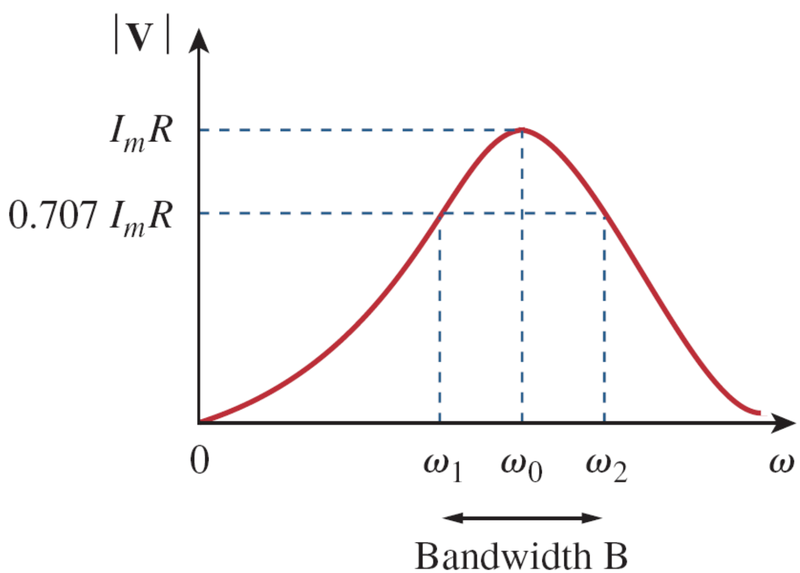
Parallel Resonance



$$\mathbf{Y} = H(\omega) = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$\mathbf{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

At resonance: $\omega C - \frac{1}{\omega L} = 0$ $\Rightarrow$ $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/s



- The voltage |V| as a function of frequency.
- At resonance, the parallel *LC* combination acts like an open circuit, so that the entire current flows through *R*.

Parallel Resonance (contd.)

- For parallel resonance:

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

- Half-power frequencies in terms of the quality factor:

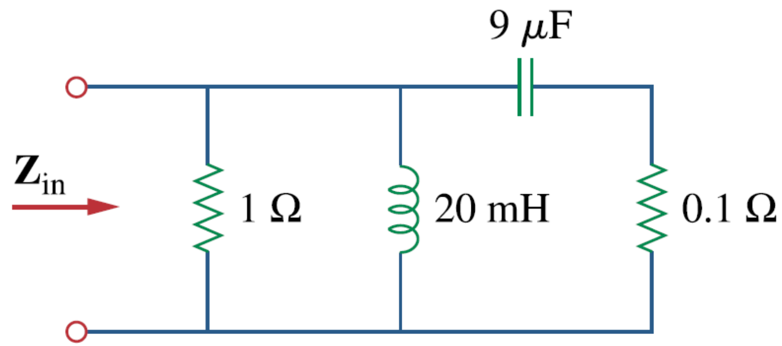
$$\omega_1 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q},$$

$$\omega_2 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

- For high-Q circuits: $\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$

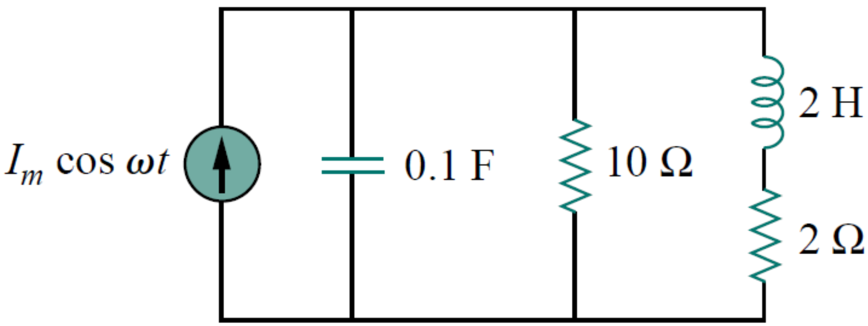
Example – 1

Find: (a) the resonant frequency ω_0; (b) $Z_{in}(\omega_0)$



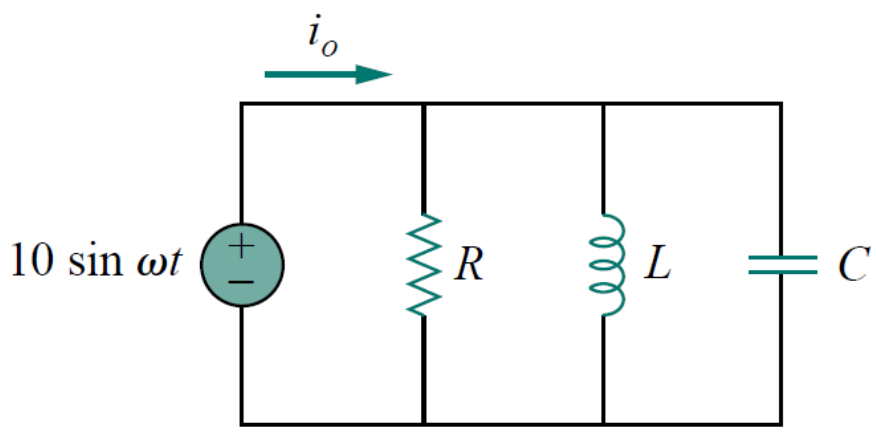
Example – 2

Determine the resonant frequency of this circuit:



Example – 3

In this parallel *RLC* circuit, let *R* = 8 kΩ, *L* = 0.2 mH, and *C* = 8 μF. (a) Calculate ω_0, *Q*, and *B*. (b) Find ω_1 and ω_2. (c) Determine the power dissipated at ω_0, ω_1, and ω_2.



Filters

A **filter** is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

- a frequency-selective device → a filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies.
- These are used in radio and TV receivers → allows the selection of one desired signal out of a multitude of broadcast signals in the environment.

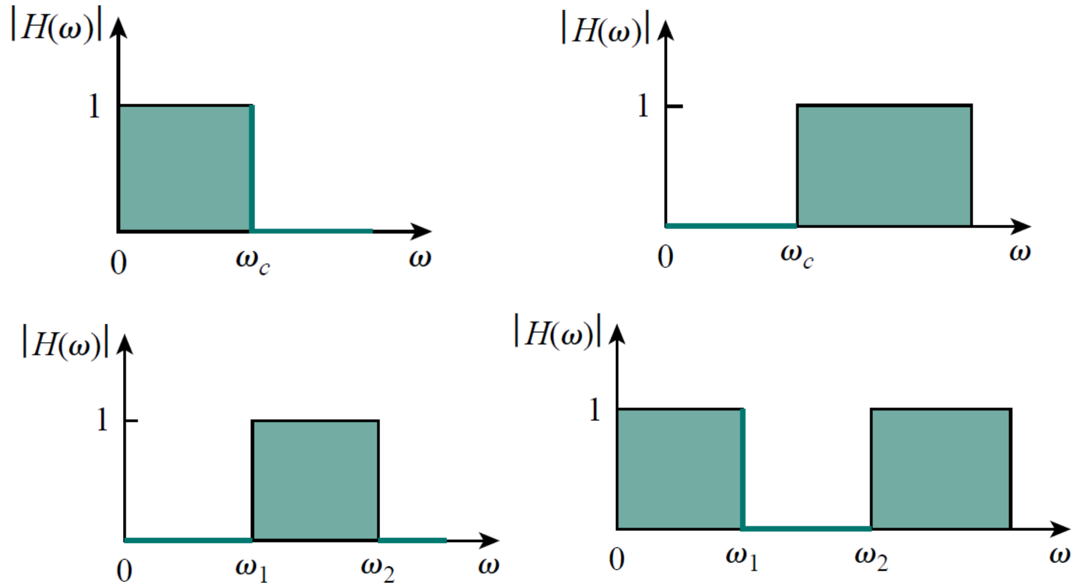
A filter is a *passive filter* if it consists of only passive elements *R*, *L*, and *C*.

It is said to be an *active filter* if it consists of active elements (such as transistors and op amps) in addition to passive elements *R*, *L*, and *C*.

Passive Filters

Filters can be classified as

- Low Pass Filter
- High Pass Filter
- Band Pass Filter
- Band Stop Filter (Band Reject/Eliminate Filter)

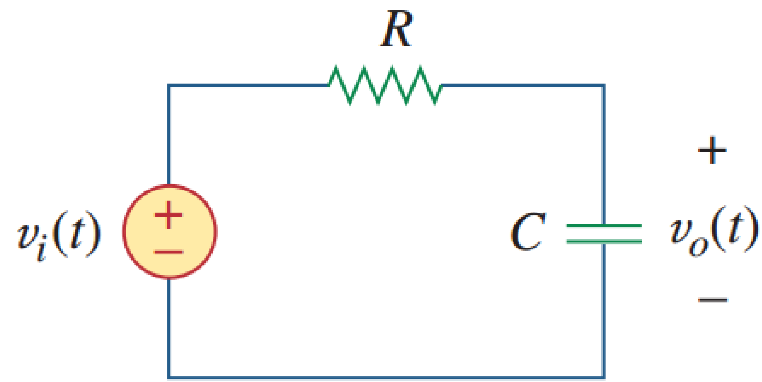


Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

ω_c is the cutoff frequency for lowpass and highpass filters; ω_0 is the center frequency for bandpass and bandstop filters.

Low Pass Filter

- LPF ideally allows lower frequencies and attenuates higher frequencies.
- A typical low pass filter is formed when the output of an *RC* circuit is taken off the capacitor.



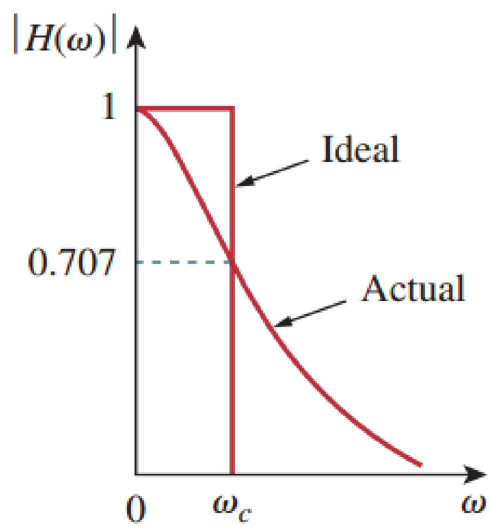
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

H(0) = 1 and H(∞) = 0

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

- ω_c is the cut-off frequency: It is a frequency at which $|H(\omega)|$ drops to 70.07% of $|H(\omega)|_{\max}$ or becomes $\frac{1}{\sqrt{2}}$ of $|H(\omega)|_{\max}$.
- So, here, ω_c can be calculated as:

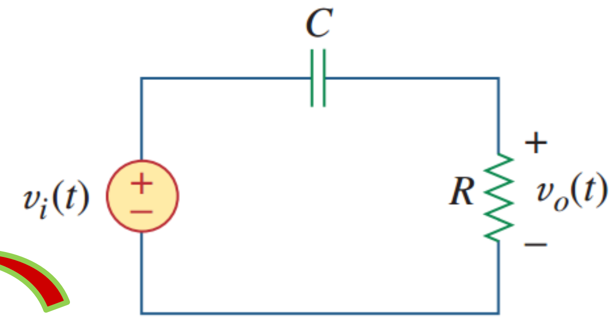
$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c = \frac{1}{RC}$$



A low pass filter can also be formed when the output of an *RL* circuit is taken off the resistor.

High Pass Filter

One of the simplest form of HPF



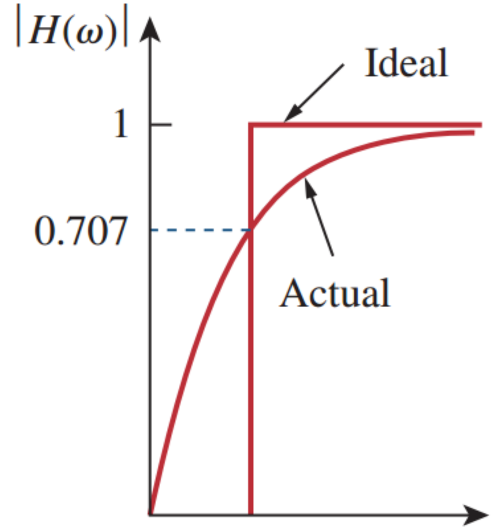
- A high pass filter is formed when the output of an *RC* circuit is taken off the resistor.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

H(0) = 0 and H(∞) = 1

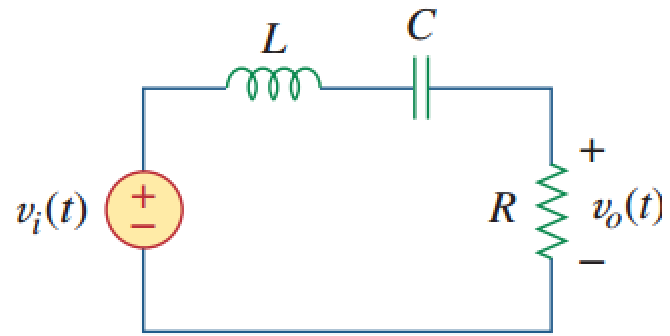
$$\omega_c = \frac{1}{RC}$$



A high pass filter can also be formed when the output of an *RL* circuit is taken off the inductor.

Band Pass Filter

The *RLC* series resonant circuit provides a band pass filter when the output is taken off the resistor



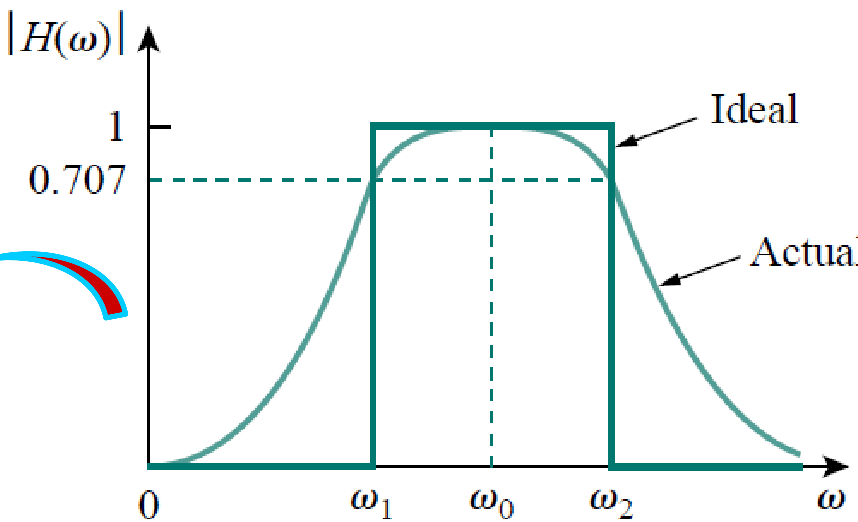
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

H(0) = 0 and H(∞) = 0

- How it is BPF ?
- Resonance Frequency, ω_0 !!!!!
- $Z_{eq} = R \Rightarrow$ Filter allows $\omega_0 \rightarrow$ BPF

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



Bandwidth of BPF = ω_2 - ω_1

Band Pass Filter

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

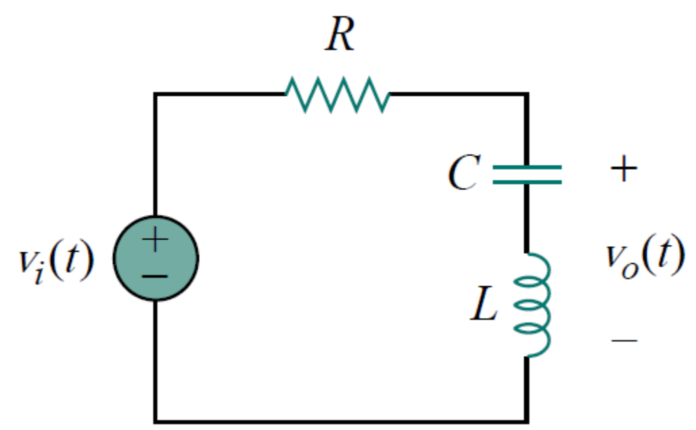
Where $\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\omega_1 \omega_1}$

A band pass filter can also be formed by cascading the low pass filter (where $\omega_2 = \omega_c$) with the high pass filter (where $\omega_1 = \omega_c$).

Band Stop Filter

A filter that prevents a band of frequencies between two designated values (ω_1 and ω_2) from passing is variably known as a *band stop, band reject*, or *notch* filter.

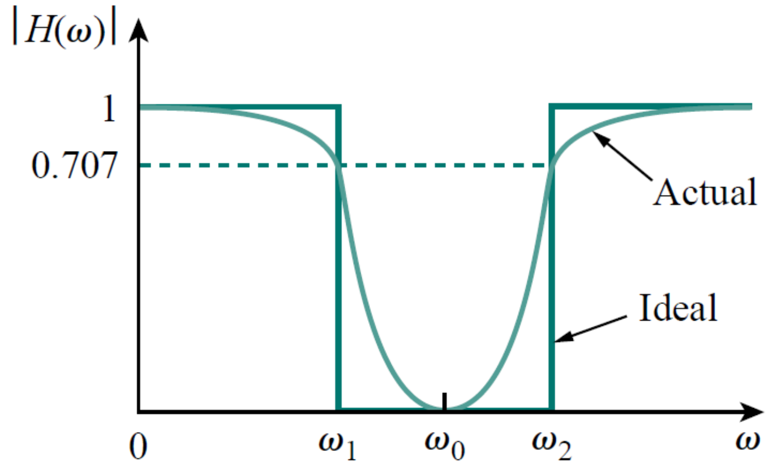
- A typical band stop filter characteristic is achieved when the **output** in the ***RLC* series resonant** circuit is **taken off the *LC* series combination**



Band Stop Filter

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

H(0) = 1, **H**(∞) = 1.



- But at resonance frequency: $v_0 = 0 \Rightarrow$ Filters blocks ω_0

Here, $\boldsymbol{\omega_0}$ is called the *frequency of rejection*, while the corresponding bandwidth (***B*** = $\boldsymbol{\omega_2}$ − $\boldsymbol{\omega_1}$) is known as the *bandwidth of rejection.*

adding the transfer functions of the band pass and the Band stop gives unity at any frequency for the same values of *R*, *L*, and *C* → results into all pass filter

Passive Filter – Summary

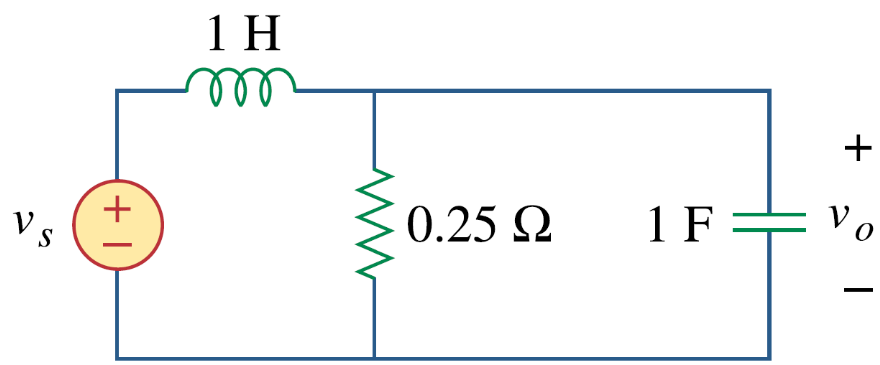
- the maximum gain of a passive filter is unity. To generate a gain greater than unity, one should use an active filter.
- There are other ways to get the types of filters.
- The filters discussed here are the simple types. Many other filters have sharper and complex frequency responses.

Example – 4

Show that a series *LR* circuit is a lowpass filter if the output is taken across the resistor. Calculate the corner frequency f_c if L = 2 mH and R = 10 k Ω .

Example – 5

Find the transfer function **V***o*/**V***s* of the circuit. Show that the circuit is a lowpass filter.



Example – 6

In a highpass *RL* filter with a cutoff frequency of 100 kHz, *L* = 40 mH. Find *R*.

Example – 7

Design a series *RLC* type bandpass filter with cutoff frequencies of 10 kHz and 11 kHz. Assuming *C* = 80 pF, find *R*, *L*, and *Q*.

Example – 8

Determine the range of frequencies that will be passed by a series *RLC* bandpass filter with *R* = 10 Ω , *L* = 25mH, and *C* = 0.4 μF. Find the quality factor.

Example – 9

Find the bandwidth and center frequency of the bandstop filter